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Towards a Thermodynamic Framework to Model Particle Crushing and Sieving

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ABSTRACT: Crushing implies the division of particles into smaller parts, and may involve a change in shape. Continuum-Based models of crushing give an average representation of elastic weakening, often in the form of a visco-plastic deformation law. Particulate Mechanics and Discrete Element Methods (DEM) are better suited to capture the loss of compression and shear strength with particle crushing. In DEM, a Representative Elementary Volume typically contains several thousands of particles, which involves high computational costs. In all of these approaches, the assembly of grains is a closed thermodynamic system. Elementary simulations of sequences alternating crushing and sieving are presented in this paper showing that the GSD obtained after each sequence is the same. However, the key point in the energy balance is the loss of mass induced by sieving, which requires additional modeling. The bases of thermodynamics of open systems subject to crushing are presented, within the framework of breakage mechanics. The proposed approach is expected to bring new insights to the modeling of friction sliding during faulting, ballast particle crushing and optimization of processes used in powder engineering.

Keywords: Crushing, Particulate Mechanics, Continuum Mechanics, Phenomenological modeling, Pore Size Distribution, Energy Optimization, Grain Size Distribution.

1. INTRODUCTION

The mechanics of particle crushing is one of the most sophisticated problems in geosciences. The topic is also of interest in many research disciplines including medicine and powder technology. Crushing can occur in faults, railways, or embankments, because of environmental conditions. Breakage could also be controlled in the engineering processes used to produce very fine particles from coarse granular materials. The problem in these processes is making finer grains provided an amount of work and energy. Energy input is directly related to the cost of the production, therefore the goal of the engineer is to minimize this input. Former studies on particle crushing include continuum mechanics, discrete element methods, or constitutive modeling of particle crushing with breakage mechanics. Several constitutive models utilizing simple curve-fitting parameters to describe the status of particle crushing were proposed, but did not account for sieving. Sieving will allow creating more friction between larger particles, because they will not be "protected" by smaller particles (which act as a shield). As a result, it is expected to gain energy in the process of making powders by sequentially removing the smaller particles being produced by crushing. In this paper, a framework is proposed to describe the evolution of a granular assembly during crushing and sieving. The model focuses on the objective size of particles in the ultimate state, and on the energy of the granular assembly during the process. Several strategies to model particle crushing are first reviewed (Section 2) before exploring the capabilities of breakage mechanics to track particle sizes during sequences of crushing and sieving (Section 3). In Section 4, bases for the corresponding thermodynamic framework are provided.

2. STATE OF THE ART: CONSTITUTIVE MODELING OF CRUSHING

2.1. Continuum-Based Models

Cyclic loads from heavy hauling trains are known to degrade and foul the ballast, which induces track settlements and densification [1]. Ballast can be viewed as an assembly of crushable particles of various sizes and shapes. Particle crushing can change the frequency of a rail track [2]. Finite Element ballast models include spring ties between blocks of elements [2]. Finite Element ballast models have been coupled to half-space Boundary Element models in order to account for the presence of inclusions (including tunnels) in the ground mass [3]. Such continuum-based approaches proved to be efficient methods to predict rail track frequency modes. However, the granular nature of ballast needs to be accounted for to model the mechanical behavior of railroad ballast subjected to cyclic loading; especially when compressive strength and failure threshold are expected to play an important role [4].

2.2. Particulate Mechanics Models

Dynamic loads promote particle rearrangement, which induces permanent deformation. This mechanism is enhanced by the occurrence of particle crushing [5]. The Discrete Element Method (DEM) has been used to model the influence of crushing on the macroscopic compressive strength of ballast material. A Representative Elementary Volume (REV) typically contains several thousands of particles (or "balls"), within "walls" representing boundary conditions [6]. The University of Illinois Aggregate Image Analyzer (UIAIA) has used a DEM software to account for the size and shape distributions of real ballast particles. A parameter has been introduced to represent the effect of particle angularity in the contact models [7]. This approach also has then been extended to ballast containing fouling agents resulting from particle abrasion [8]. Image analysis is useful to get realistic geometric descriptions of particle assemblies at a given time step or loading cycle. Nevertheless, the method does not capture the degradation process over time.

Recent DEM frameworks model abrasion as bond breakages within a cluster of bonded spherical balls. In the initial state, one cluster represents one intact ballast particle. The predicted yield stress for the agglomerates turn to be less than that for the real ballast, mainly because the spherical shape of the clusters cannot reproduce well chains of forces in the degraded ballast [9]. The framework has been improved by introducing clumps, defined as entities of overlapping balls, "rigid internally and deformable at the external boundary" [10]. Clumps resemble real ballast particles much more than spheres do, and provide more realistic predictions [11, 1, 12]. The method proves to reproduce well ballast degradation observed in cyclic triaxial tests, where asperity fracture dominates [10].

2.3. Breakage Mechanics

Einav proposed a theoretical model [13] in which the evolution of the crushing process in a granular assembly is tracked with a fractional breakage parameter B, defined as:

$$B = \frac{p_0(d) - p(d)}{p_0(d) - p_u(d)} = \frac{F_0(d) - F(d)}{F_0(d) - F_u(d)} \tag{1}$$

In which $p(d)$ is the current Grain Size Distribution (GSD), $p_0(d)$ is the initial GSD of the soil sample, $p_u(d)$ is the ultimate GSD (expected at the end of the crushing process), $F(d)$ is the current cumulative GSD, $F_0(d)$ is the initial cumulative GSD, and $F_u(d)$ is the ultimate cumulative GSD. Therefore, Eq. (1) allows to compute the current and cumulative GSDs of a granular material during crushing as long as the initial and ultimate GSDs are given. B is independent of the grain size, i.e. the evolution of crushing occurs uniformly through the GSD until the cumulative GSD reaches the ultimate distribution. Several assumptions can be made regarding the ultimate GSD of particles after extreme crushing. Earlier studies [14] considered that all particles, regardless of their initial size, would completely crush until all particles become finer than an arbitrary size [14]. The ultimate "tracked" size was 0.074 mm, which means that after extreme crushing, all crushed particles would finally pass Sieve #200. Furthermore, extensive studies consistently showed that the ultimate GSD couldn't be any arbitrary distribution: it is bounded by a specific distribution, i.e. it is not possible to crush a granular assembly to a powder with indefinitely smaller particles. There is a limit below which the work needed for crushing cannot break particles, which results in a specific ultimate distribution [15]. There is still no consensus on what should be the ultimate GSD after crushing, but in most practical applications, a fractal distribution is assumed.

3. EVOLUTION OF GSD DURING CYCLES ALTERNATING CRUSHING AND SIEVING – A PARAMETRIC STUDY

As observed experimentally by [16], smaller particles tend to surround larger particles and protect them against breakage during the crushing process. As it is shown in Fig. (1), smaller particles surrounding larger particles start to break before larger particles. Therefore, smaller particles are more prone to be crushed than larger particles. This "Shielding effect" makes an upper-bound for the ultimate GSD because even after extreme crushing of particles, larger grains get protected by smaller ones and resist against breakage. Therefore, to reach an ultimate distribution dominated by smaller sizes, it is expected that removing regularly smaller particles by sieving should make crushing more energy-efficient. It is proposed herein to study the evolution of the GSD of a granular assembly subjected to cycles alternating crushing and sieving. The purpose of the following simulations is to illustrate microstructure evolution during the process of crushing/sieving, and to identify GSD parameters useful to compute the energy of the system during the process.

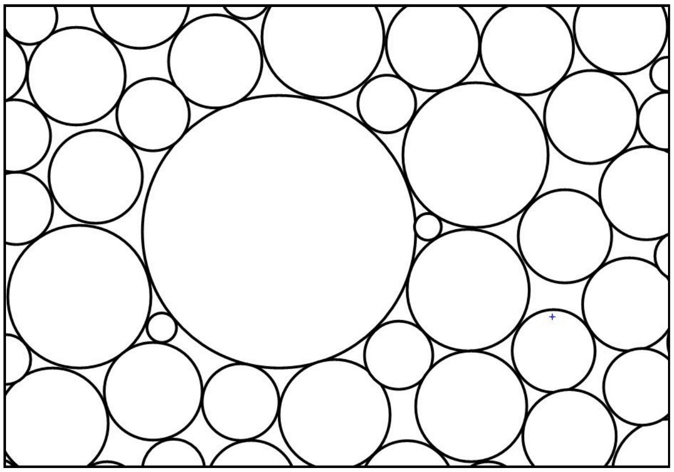

Fig. 1. Shielding Effect in a Granular Assembly.

A parametric study was performed, with various initial and ultimate GSDs, with and without sieving. Particle sizes during the process are tracked with the parameter defined in Eq. (1). In the following, breakage mechanics [13] is used as a reference model. The purpose of the simulations is: (1) to test whether the model proposed in [13] properly predicts the evolution of crushing for various GSDs – not only the common "well-graded initial GSD – fractal ultimate GSD", and (2) to test the ability of breakage mechanics frameworks to model processes including sieving (Tab.1).

Table 1. Three different cases with different ultimate and initial distributions. Case 3 includes a sieving process.

Case #	Initial GSD	Ultimate GSD	Sieving
1	Gap-graded	Fractal	No
2	Well-graded	Ultimate with plateau	No
3	Well-graded	Fractal	Yes

3.1. Case 1: Gap-Graded Initial GSD with Ultimate Fractal Distribution

First, a gap-graded soil sample is considered for the initial GSD. Therefore, a range grain sizes is missing from the sample. The ultimate distribution is assumed to be fractal. An ultimate fractal distribution can be approximated as:

$$F_u(d) = \left(\frac{d}{d_M}\right)^{3-\alpha} \tag{2}$$

Where, d is the grain size, d_M is the maximum grain size in the soil sample, and α is a parameter defining the shape of the distribution. For most practical purposes, α is considered equal to 2.6 [13], which is the choice made in the study below.

The evolution of crushing is tracked by using Eq. (1). The GSD (obtained from Eq. (1)) for different values of B (equal to 0.2, 0.5 and 0.8) are presented in Fig.2. Note that in the figures, instead of the actual grain size d, a normalized grain size (d_{norm}) is used, which is defined as:

$$d_{norm} = \frac{d}{d_M} \tag{3}$$

Using Eq. (3), Eq. (2) is rewritten as:

$$F_u(d) = \left(d_{norm}\right)^{3-\alpha} \tag{4}$$

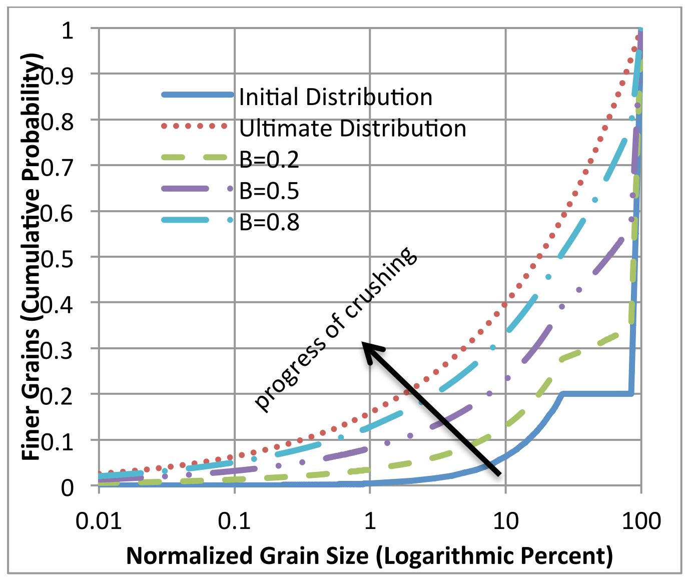

Fig. 2. Case 1: Crushing of a gap-graded granular material using breakage mechanics parameter B.

Eq. (1) inherently assumes that the time evolution from initial to ultimate GSD is linear. In this specific gap-graded case, the initial GSD is missing particle sizes in the range of 30% to 80% for the normalized grain size d_{norm} ("initial distribution" in Fig.2). It is reasonable to assume that in the process of crushing, some larger particles break into smaller particles, thus filling the gap in the initial gap-graded distribution. Fig. 2 shows that using the model presented in [13] (based on Eq. (1)) allows to predict these trends: breakage mechanics seems suitable to predict the crushing of gap-graded granular materials with a fractal ultimate GSD.

3.2. Case 2: Well Graded Initial GSD and a Plateau in the Ultimate Distribution

To check the effect of the assumption made on the ultimate GSD, we assume that the ultimate GSD is not a fractal distribution. An ultimate distribution with largest particle size d_{Mu}, smaller than the largest particle size in the initial GSD d_{Mi} is considered (Fig. 3). This is to account for the disappearance of the largest particles during the process.

We assume that all particles with normalized size greater than 50% of d_{Mi} will break to smaller particles. Crushing is simulated for the following values of the B parameter: 0.4, 0.7, and 0.9. Using Eq. (1), the current grain size cumulative distribution is obtained and the graphs are shown in Fig.3. It is expected that the particles with normalized size larger than 50% would gradually crush and disappear, making the current GSD closer and closer to the ultimate GSD. As it is shown in Fig. 3, the part of current GSD that contains grains with normalized size larger than 50% is properly approaching to the plateau in the ultimate GSD. Therefore, Eq. (1) can also be used when a general ultimate distribution and not specifically a fractal distribution is considered.

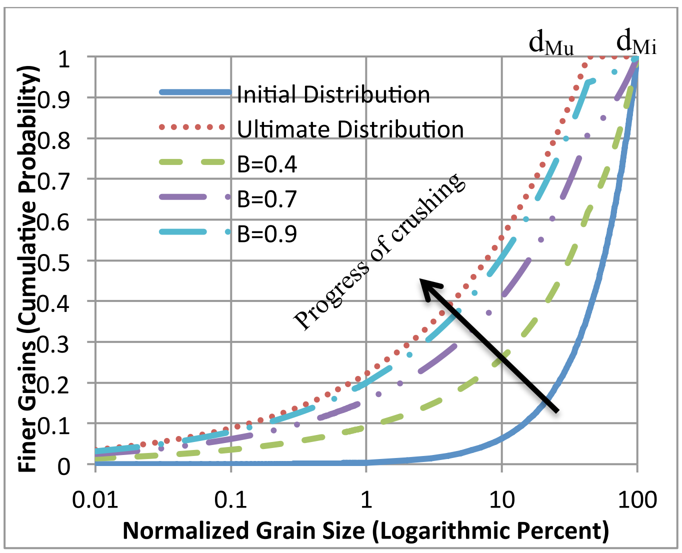

Fig. 3. Case 2: Crushing of a well-graded granular material using breakage parameter B. d_{Mu} and d_{Mi} are normalized largest particle sizes in the ultimate and initial GSD, respectively.

3.3. Case 3: Sequential Evolution of Crushing Including Sieving

We now consider a well-graded soil sample with a known initial GSD and a fractal ultimate distribution. We load the soil sample until the theoretical value of B becomes equal to 0.5. Using Eq. (1), the current cumulative GSD is estimated and we refer to this GSD as "step 1". Initial, ultimate, and current GSDs are plotted together in Fig. 4a. After B becomes equal to 0.5 (i.e. after the GSD has moved half way between the given initial and ultimate GSDs), smaller particles are removed to avoid the shielding effect mentioned earlier.

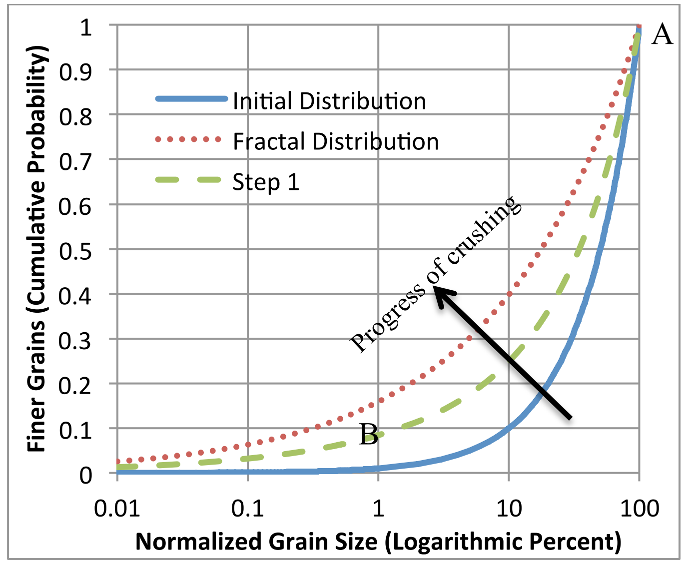

Fig. 4a. Case 3. Step 1 of crushing/sieving.

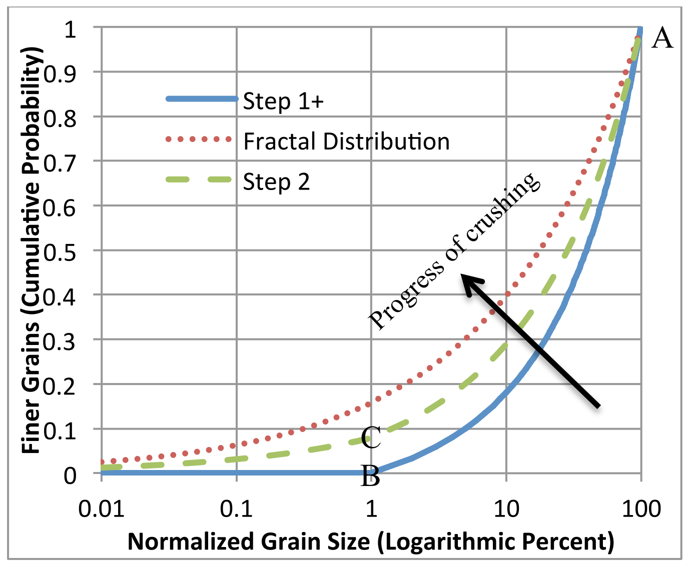

Fig. 4b. Case 3. Step 2 of crushing/sieving.

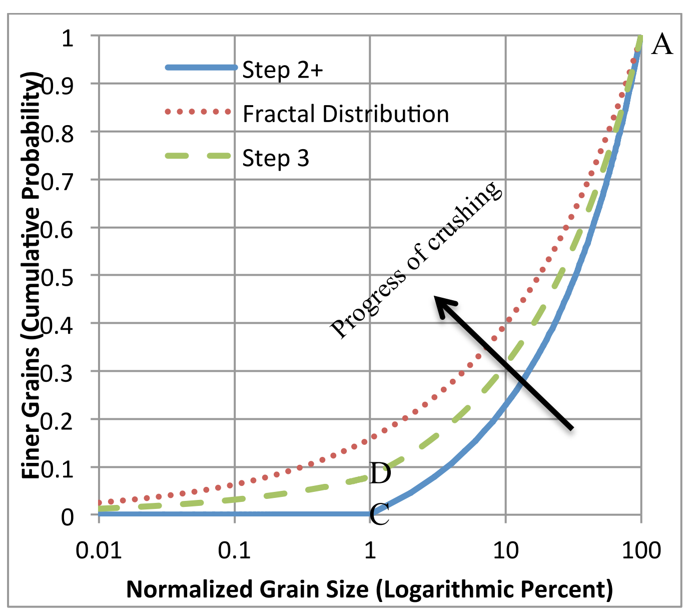

Fig. 4c. Case 3. Step 3 of crushing/sieving.

To illustrate the procedure, a simulation is performed in which all particles smaller than 1% of the maximum grain size (d_{norm} smaller than 1%) are removed by sieving. The new cumulative distribution can be obtained after removing the smaller particles and it is shown in Fig. 4b., called step 1^{+} (the superscript "+" means "removal of the smaller particles after the first step of loading"). A second crushing phase is simulated, until parameter B (defined in Eq. (1)) becomes equal to 0.5. The new current GSD is called step 2 and is plotted in Fig. 4b. along with the ultimate fractal and step 1^{+} distributions. Then, particles smaller than 1% of the maximum grain size (d_{norm} smaller than 1%) are removed again, and the new cumulative distribution called step 2^{+} is plotted in Fig. 4c. The same procedure is repeated for the next steps of crushing (i.e. step 4 and beyond).

As it can be seen in Fig. 4b and Fig. 4c, the shape of the new cumulative distribution does not significantly change, and the same shape is obtained if the process is simulated for steps beyond step 3. However, the mass of particles present in the system changes during the process (sieving implies that the thermodynamic system is open). A better way to illustrate the steps of crushing is to consider the mass of the system in the model. For this purpose, we define a normalized mass:

$$F_m(d) = F(d) \cdot \frac{M_n}{M_I} \tag{5}$$

where, $F_m(d)$ is the normalized mass of the system for the current cumulative grain size distribution, $F(d)$ is the cumulative grain size distribution, M_n is the total mass of the system in the n_{th} step of the crushing/sieving process, and M_I is the total initial mass of the system (before any crushing/sieving occurs). In each step of crushing, the mass of the smaller particles should be removed from the system, in order to simulate sieving. As a result, the value of M_n / M_I should approach zero as sequences including sieving proceed. Simulation of "Case 3" (with sieving), with account of the mass change, is presented in Fig. 5.

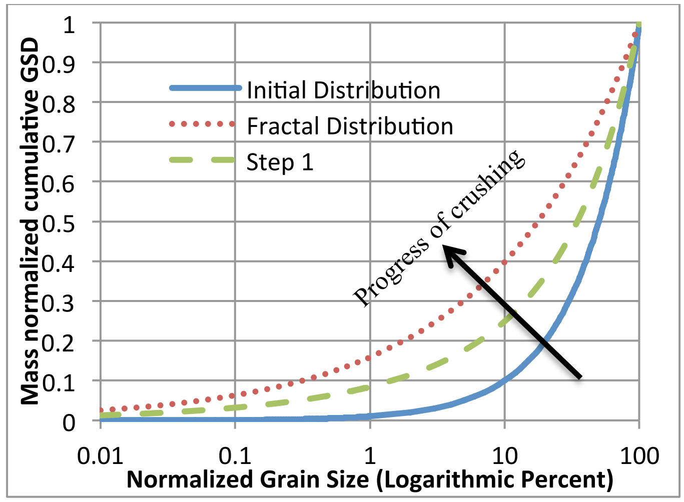

Fig. 5a. Case 3 with updated mass. Step 1 of crushing/sieving.

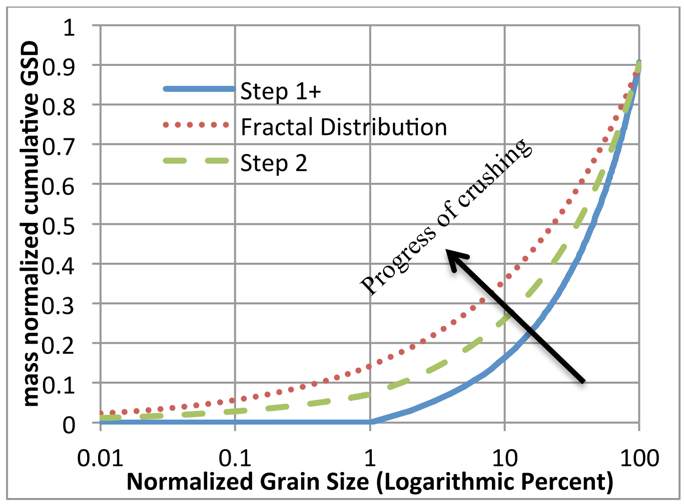

Fig. 5b. Case 3 with updated mass. Step 2 of crushing/sieving.

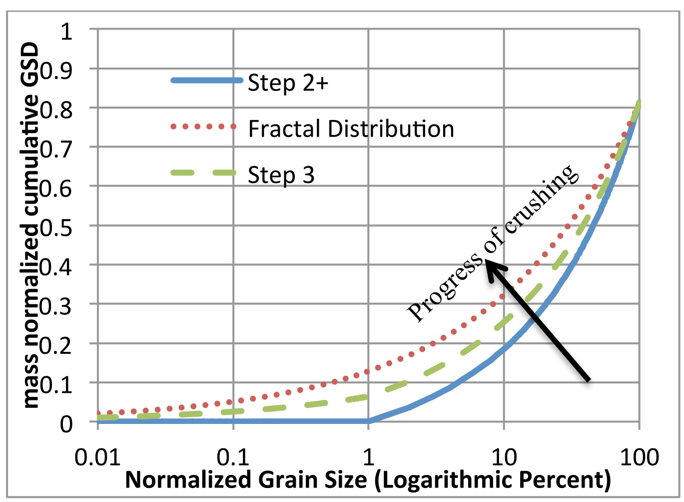

Fig. 5c. Case 3 with updated mass. Step 3 of crushing/sieving.

Fig 5. Presentation of three steps of crushing/sieving of granular materials. Mass normalized cumulative GSD, $F_m(d)$ is used for the vertical axis.

After the first step, particles with normalized size less than 1% are sieved. In Fig. 5a., the value of the normalized mass defined in Eq. (5) which corresponds to a normalized size d_{norm} equal to 1% is approximately 9% (see "Step 1" plot). Therefore, 9% of the mass of the system is removed in "Step 1" sieving. As a result, in Fig 5b showing Step 2, the maximum normalized mass is 91% because 9% of the soil has already been sieved in the previous step and is not in the system anymore. In step 2, again particles with normalized size less than 1% are removed. In Fig. 5b. and for "Step 2" curve, we read the value of mass normalized cumulative GSD on the vertical axis according to normalized size equal to 1%, which is approximately equal to 8% (8% of the initial total mass). Therefore, the soil mass again reduces by 8% for "step 3". As a result in Fig. 5c, for "step 3", the maximum normalized mass is 83%: the mass of the system was shrunk by 9% and 8% in steps 1 and 2, respectively.

Although the probability distribution is relatively constant for all steps (Fig. 4), the mass of the system is reducing and if the number of steps approaches to infinity, the mass of the system decreases continuously (i.e. all crushed particles will be removed from the system by sieving).

The sieving process could also be done after completion of crushing at each step (for B equal to one). However, to minimize the amount of energy spent to reach a target GSD, it would be more cost effective to sieve smaller particles before finishing a complete step (for B less than one, like in this parametric study). In so doing, finer particles could be removed before the shielding effect completely protects larger particles. However, the optimal value of B is not known. It is proposed to determine this value by minimizing the total energy, which is given to the system to reach a target GSD. For this purpose, in the next section, we introduce the bases of thermodynamic rules governing the evolution of GSD during crushing/sieving sequences.

4. EVOLUTION OF GSD DURING CRUSHING/SIEVING SEQUENCES: THERMODYNAMIC BASES

First, some statistic rules used further to construct the thermodynamic framework are recalled. Consider a variable A (e.g., deformation, stiffness, energy or any internal variable). The following definitions are used to characterize probabilistic averages over the GSD function:

$$\bar{A} \equiv \langle A \rangle = \int_{d_m}^{d_M} A(d)\, p(d)\, \mathrm{d}d \quad (6)$$

$$\bar{A}_0 \equiv \langle A \rangle_0 = \int_{d_m}^{d_M} A(d)\, p_0(d)\, \mathrm{d}d \quad (7)$$

$$\bar{A}_u \equiv \langle A \rangle_u = \int_{d_m}^{d_M} A(d)\, p_u(d)\, \mathrm{d}d \quad (8)$$

Where, $\bar{A}$ is the average of variable A over the current GSD, $\bar{A}_0$ is the average of variable A over the initial GSD, $\bar{A}_u$ is the average of the variable A over the ultimate GSD, d_M is the maximum grain size in the soil sample, and d_m is the minimum grain size in the soil sample. Using Eq. (1) and eqs. (6), (7) and (8), we have:

$$\bar{A} = \bar{A}_0 (1 - B) + \bar{A}_u B \quad (9)$$

The dissipation inequality writes:

$$\tilde{W} = \delta\Psi + \tilde{\Phi}, \quad \tilde{\Phi} \geq 0 \quad (10)$$

Where Ψ and $d\Psi$ are the Helmholtz free energy and its increment, respectively; $\tilde{\Phi}$ is the non-negative increment of energy dissipation; $\tilde{W}$ is the increment of mechanical work done on the boundaries of the system. The work input is equal to strain energy (Principle of Virtual Work):

$$\tilde{W} = \sigma : \delta\varepsilon \quad (11)$$

where σ and ε are the stress and strain tensors that apply to the boundaries of the REV, $\delta\varepsilon$ while is the increment of strain. Using a uniform crushing model [13] and with $A = \Psi$ we can get the following equations for each crushing/sieving sequence:

$$\bar{\Psi}_1 = \langle \hat{\Psi} \rangle_1 = \langle \hat{\Psi} \rangle_0 (1 - B_1) + \langle \hat{\Psi} \rangle_u B_1 = \bar{\Psi}_0 (1 - B_1) + \bar{\Psi}_u B_1$$

$$\bar{\Psi}_2 = \langle \hat{\Psi} \rangle_2 = \langle \hat{\Psi} \rangle_{1^+} (1 - B_2) + \langle \hat{\Psi} \rangle_u B_2 = \bar{\Psi}_{1^+} (1 - B_2) + \bar{\Psi}_u B_2$$

$$\bar{\Psi}_N = \langle \hat{\Psi} \rangle_N = \langle \hat{\Psi} \rangle_{(N-1)^+} (1 - B_N) + \langle \hat{\Psi} \rangle_u B_N = \bar{\Psi}_{(N-1)^+} (1 - B_N) + \bar{\Psi}_u B_N$$

(12)

in which B_n is the crushing parameter for step n of the crushing/sieving process.

We postulate the following Helmholtz free energy density function in particles with size d [13]:

$$\bar{\Psi} = \bar{\Psi}(d, \varepsilon) = f_\psi(d)\, \Psi_r(\varepsilon) \quad (13)$$

In which $f_\psi(d)$ is the energy split function.

$$\bar{\Psi}_n\left(d,\varepsilon\right) \equiv \psi_{r,n}\left(\varepsilon\right)\underbrace{\left[\left(1-B_n\right)m_{(n-1)^+}+B_n m_u\right]}_{\left\langle f_{\psi,n}(d)\right\rangle} \quad (14)$$

At the beginning of the first sequence of crushing/sieving (n=1), the mean value of the energy split function (that quantifies the energetic contribution of each grain size to the granular assembly) is:

$$m_0 = \int_{d_m}^{d_M} f_\psi\left(\Delta\right) p_0\left(\Delta\right) d\Delta = \left\langle f_\psi\left(d\right)\right\rangle_0 \quad (15)$$

For later sequences, i.e. for n > 1

$$m_{(n-1)^+} = \int_{d_s}^{d_M} f_\psi\left(\Delta\right) p_{(n-1)^+}\left(\Delta\right) d\Delta = \left\langle f_\psi\left(d\right)\right\rangle_{(n-1)^+} \quad (16)$$

where d_s is the size of the sieve opening (assumed to be the same at each sequence for simplicity). Note that $d_s = d_m$ because all particles smaller than d_s are removed by sieving.

The challenging part of the modeling consists in determining the relationship between the energy of the granular assembly at the end crushing and the energy of the assembly at the end of sieving, within a sequence of crushing/sieving. The loss of energy of the system between the end of crushing and the end of sieving phases writes:

$$\Delta\bar{\Psi} = \bar{\Psi}_{crush}\left(\varepsilon,d\right) - \bar{\Psi}_{siev}\left(\varepsilon,d\right) \quad (17)$$

In which the free energy of the assembly at the end of crushing (resp. at the end of sieving) is noted as:

$$\bar{\Psi}_{crush}\left(\varepsilon,d\right) \text{ (resp. } \bar{\Psi}_{siev}\left(\varepsilon,d\right)\text{)} \quad (18)$$

And they are defined as:

$$\begin{aligned} \bar{\Psi}_{crush}\left(\varepsilon,d\right) &= \int_{d_m}^{d_M} f_\psi\left(\Delta\right)\psi_r\left(\varepsilon\right) d\Delta \\ \bar{\Psi}_{siev}\left(\varepsilon,d\right) &= \int_{d_s}^{d_M} f_\psi\left(\Delta\right)\psi_r\left(\varepsilon\right) d\Delta \end{aligned} \quad (19)$$

According to Eq. (14), the statistical average of the Helmholtz free energy for the n_{th} sequence of crushing is:

$$\delta\Psi_n \equiv \frac{\delta\Psi_{rn}\left(\varepsilon\right)}{\delta\varepsilon}\left[\left(1-B_n\right)m_{(n-1)^+}+B_n m_u\right]\delta\varepsilon + \Psi_{rn}\left(\varepsilon\right)\left(m_u - m_{(n-1)^+}\right)\delta B_n$$

(20)

Determining the differential expressed in Eq. (20) should lead to the determination of the loss of energy, which writes:

$$\Delta\bar{\Psi} = \int_{d_m}^{d_s} f_\psi\left(\Delta\right)\psi_r\left(\varepsilon\right) d\Delta \quad (21)$$

When sieving is finished, a new crushing step imposes $B_n=0$. The new initial conditions straws of crushing provide the expression of the initial $\psi_{r,n}\left(\varepsilon\right)$:

$$\psi_{r,n}\left(\varepsilon\right) = \frac{\Psi\left(B \equiv 0\right)}{m_0} \quad (22)$$

To update the mean value of the split function (used in Eq. (15)) and Eq. (20), the probability density function of the GSD needs to be updated. The updated probability grain size distribution after sieving of particles smaller than d_s writes:

$$p_{(n-1)^+}\left(\Delta\right) = \frac{p_{(n-1)}\left(\Delta\right)}{1-\int_{d_m}^{d_s} p_{(n-1)}\left(d\right)\mathrm{d}d} = \frac{p_{(n-1)}\left(\Delta\right)}{1-F_{(n-1)}\left(d_{msn}\right)}$$

for $d_s < \Delta < d_M$, (23)

and:

$$p_{(n-1)^+}\left(\Delta\right) = 0 \text{ for } \Delta < d_s \, or \, \Delta > d_M \quad (24)$$

where, $p_{(n-1)}\left(\Delta\right)$ and $F_{(n-1)}\left(\Delta\right)$ are the GSD and cumulative GSD functions for the soil sample before sieving in the nth step, respectively.

Updating the probability density function is enough to update the value of m_{n-1} in Eq. (20) because the energy split function $f_\psi\left(d\right)$ is given as part of the assumptions for the model [13]. For instance, for disks in 2D:

$$f_\psi\left(d\right) = \frac{d}{\bar{d}_0} \quad (25)$$

and for three-dimensional spheres:

$$f_\psi\left(d\right) = \frac{d^2}{J_{20}} \quad (26)$$

Where

$$J_{20} = \int_{d_m}^{d_M} d^2 p\left(d\right) \mathrm{d}d \tag{27}$$

5. CONCLUSIONS

The bibliographic review presented in this paper shows that the particle-scale behavior cannot be ignored in the modeling of crushed granular assemblies and that it is necessary to better relate energy dissipated by crushing at the grain scale and at the bulk scale. The breakage mechanics model proposed by Einav [13] relates the two scales within a consistent thermodynamic framework for closed systems. The parametric study on initial and ultimate Grain Size Distributions (GSD) illustrates the capabilities of the model to track particle sizes during the process of crushing. However, phenomenological modeling of crushing for open systems remains a challenge. Simulations of sequences alternating crushing and sieving show that the GSD obtained after each sequence is the same. The key point in the energy balance is the loss of mass induced by sieving. Starting form Einav's model of uniform crushing, the bases of thermodynamics of open systems subject to crushing are explained. In its present form, the framework requires assumptions on the grain shape ratios and separation of variables in the expression of the bulk free energy. These assumptions will be investigated more in depth in future work. The proposed approach is expected to bring new insights in the modeling of friction sliding during faulting, ballast particle crushing, and optimization of processes used in powder engineering.

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